# CS 61A Lecture 11

# Announcements

# Box-and-Pointer Notation

# The Closure Property of Data Types

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- A method for combining data values satisfies the *closure property* if:

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- Hierarchical structures are made up of parts, which themselves are made up of parts, and so on

Lists can contain lists as elements (in addition to anything else)

# Box-and-Pointer Notation in Environment Diagrams

Interactive Diagram

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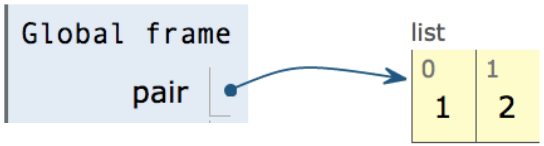


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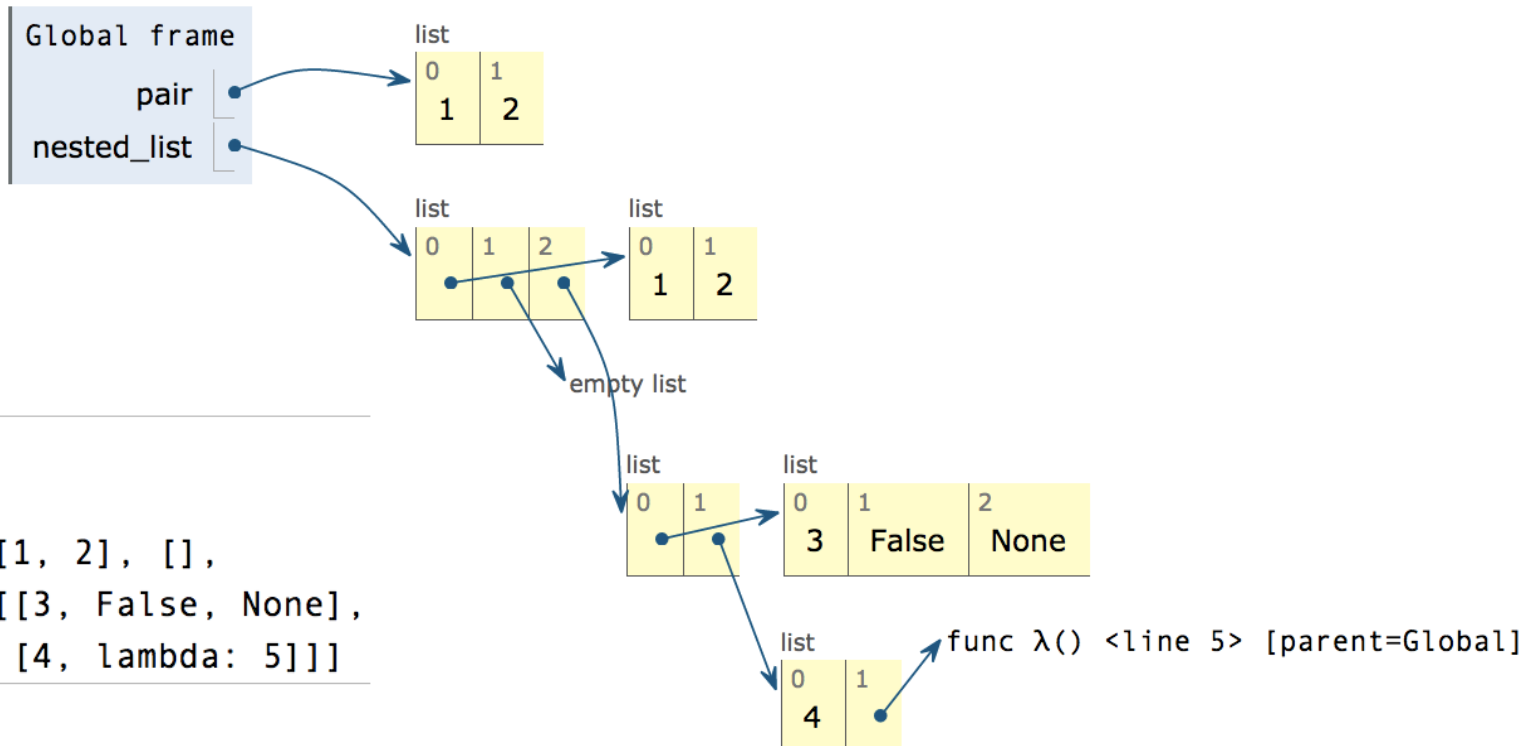



```
1  pair = [1, 2]
2
3  nested_list = [[1, 2], [],
4                 [[3, False, None],
5                  [4, lambda: 5]]]
```

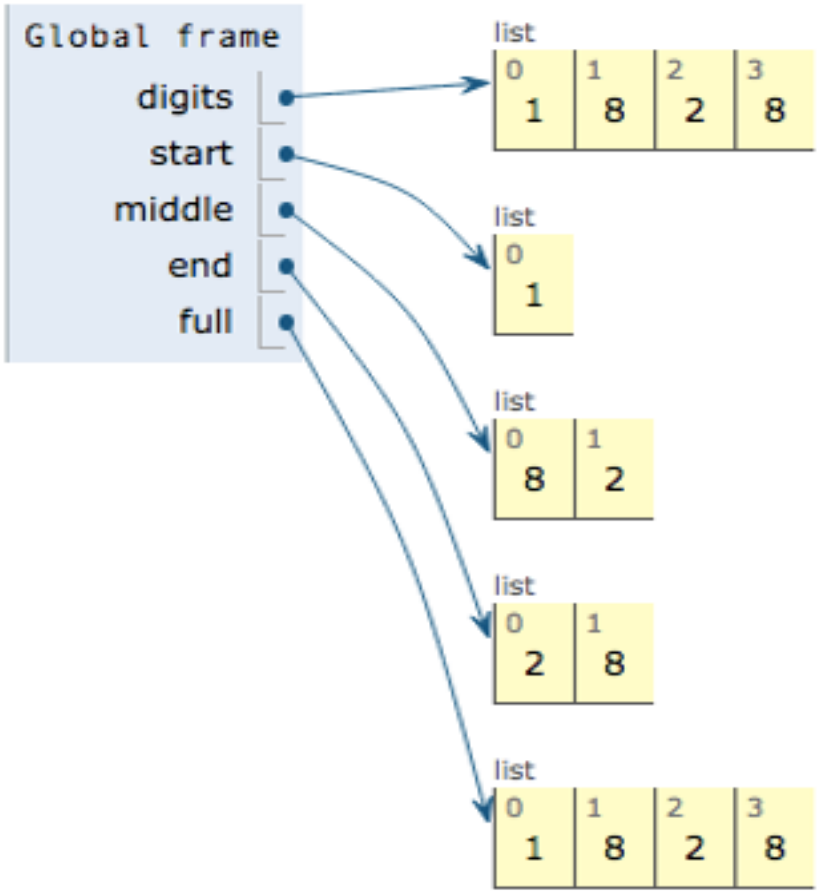
Interactive Diagram

# Slicing

(Demo)

# Slicing Creates New Values

```
1 digits = [1, 8, 2, 8]
2 start = digits[:1]
3 middle = digits[1:3]
4 end = digits[2:]
5 full = digits[:]
```

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# Processing Container Values

# Sequence Aggregation

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  With a single iterable argument, return its largest item.
  With two or more arguments, return the largest argument.

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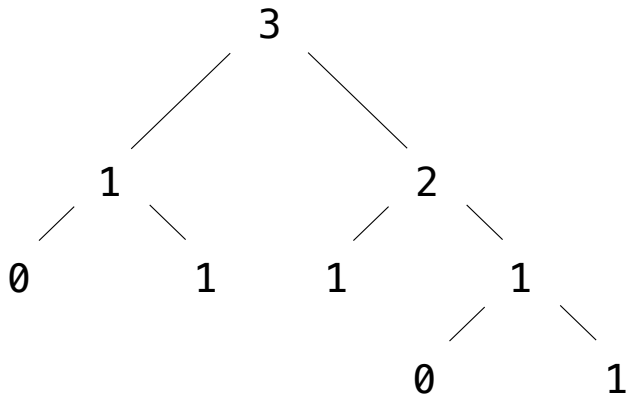
- **all**(iterable) -> bool

  Return True if bool(x) is True for all values x in the iterable.
  If the iterable is empty, return True.

# Trees

# Tree Abstraction



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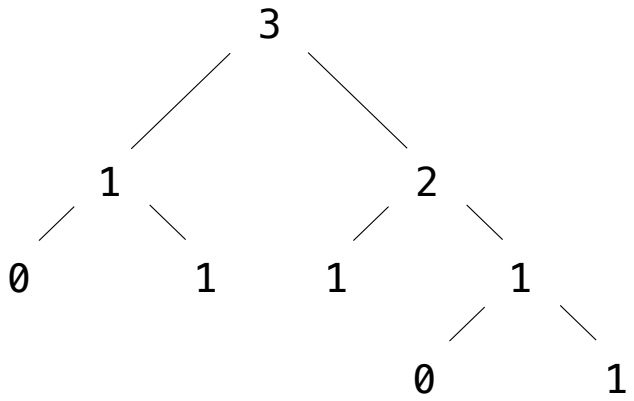


**Recursive description (wooden trees):**

**Relative description (family trees):**

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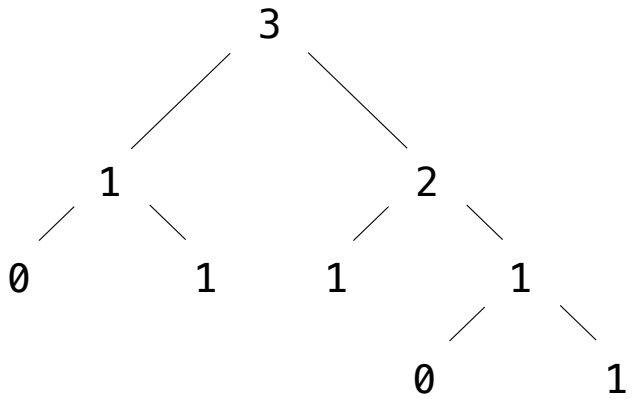


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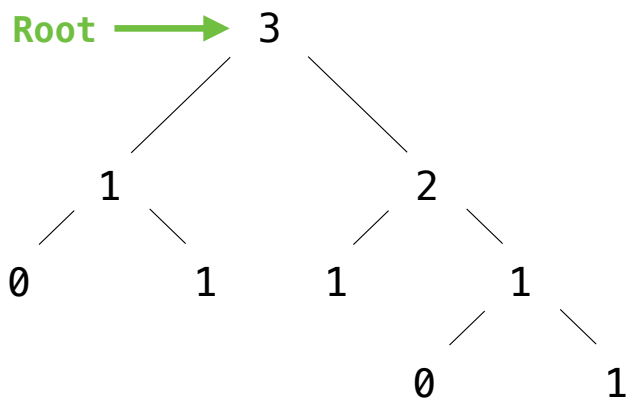


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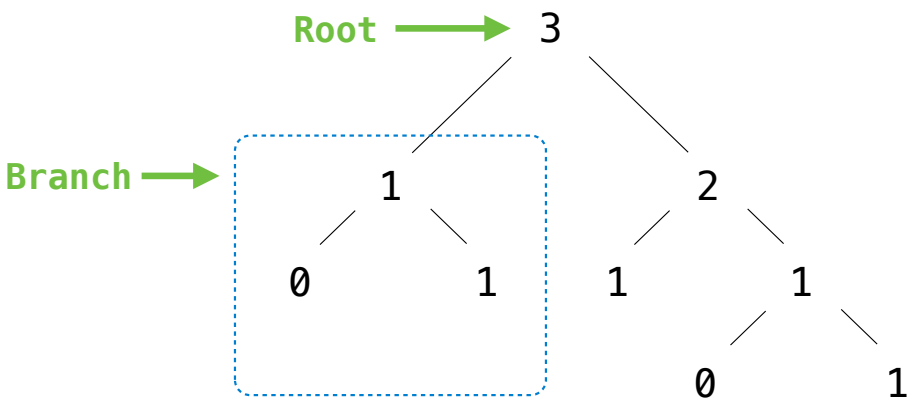


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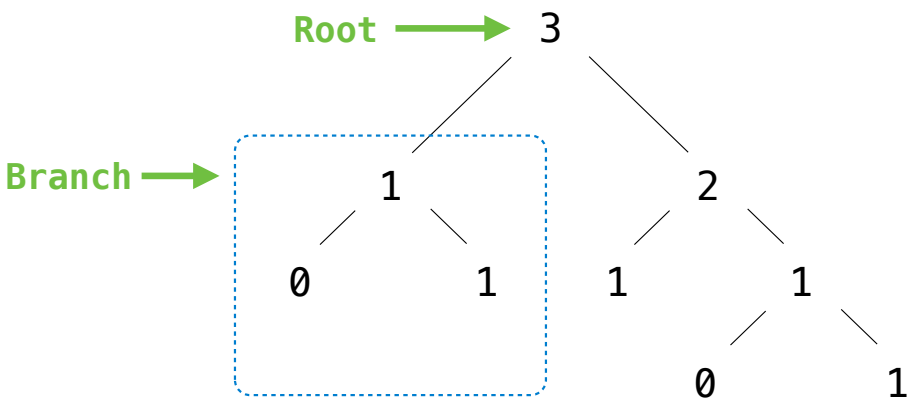


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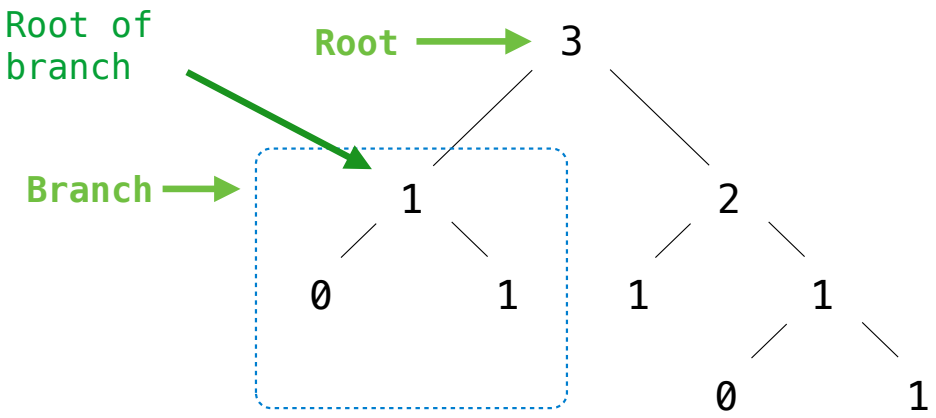


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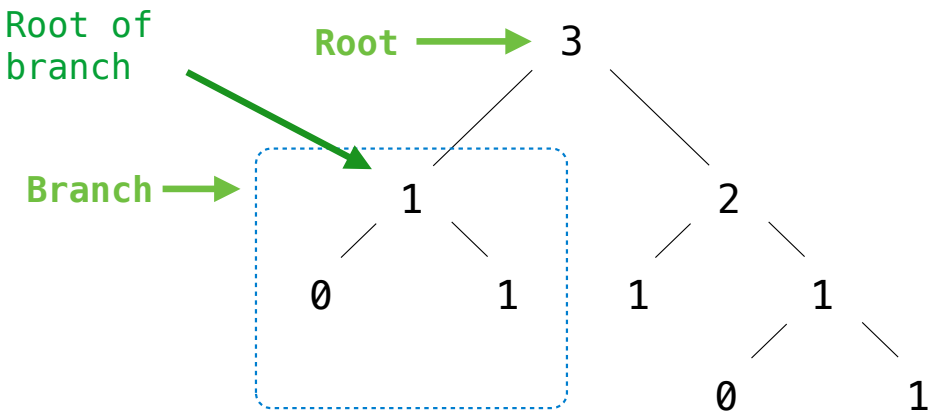


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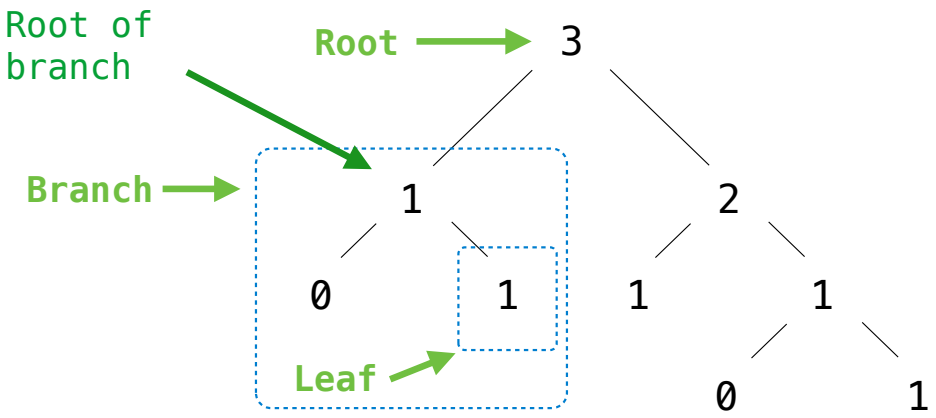


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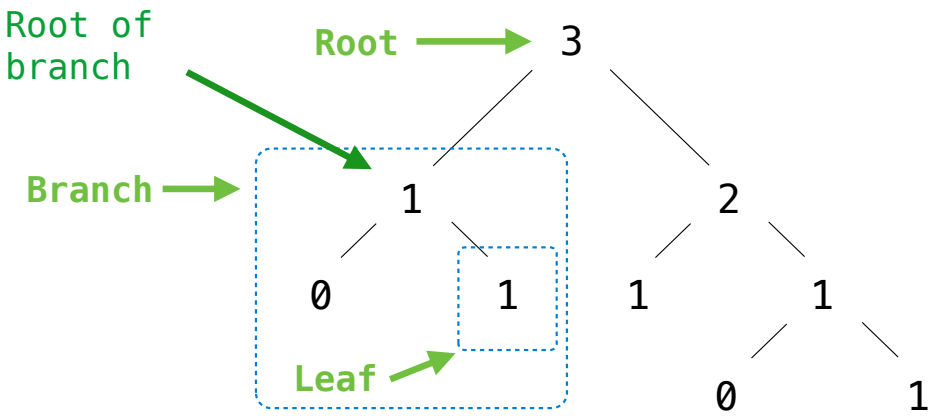


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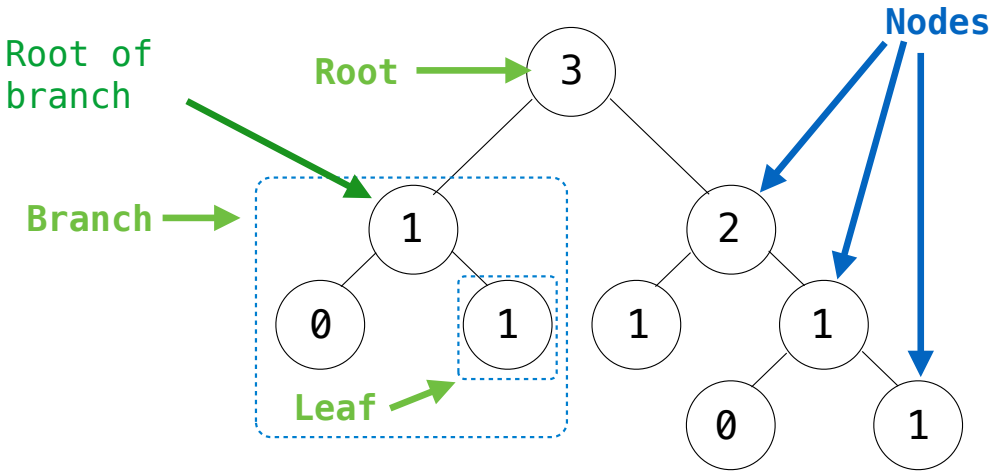


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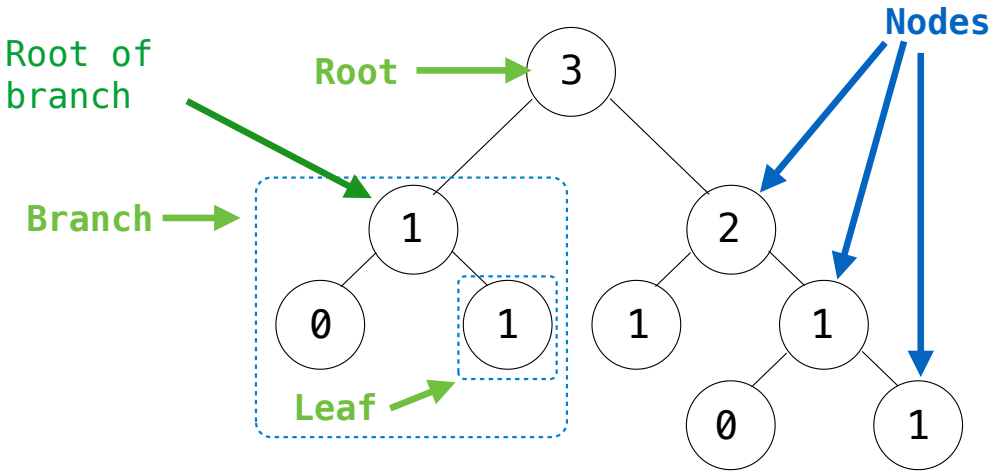


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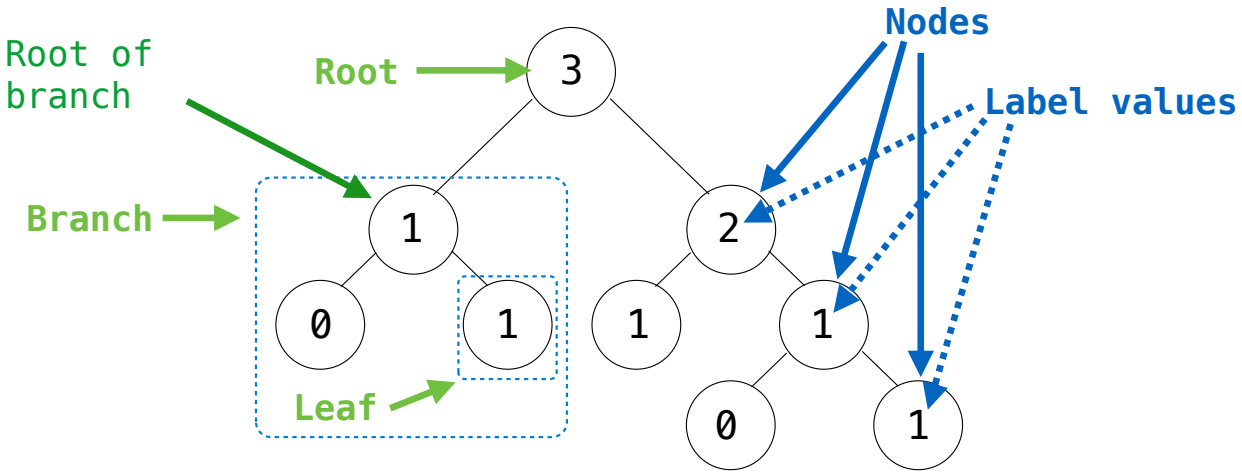


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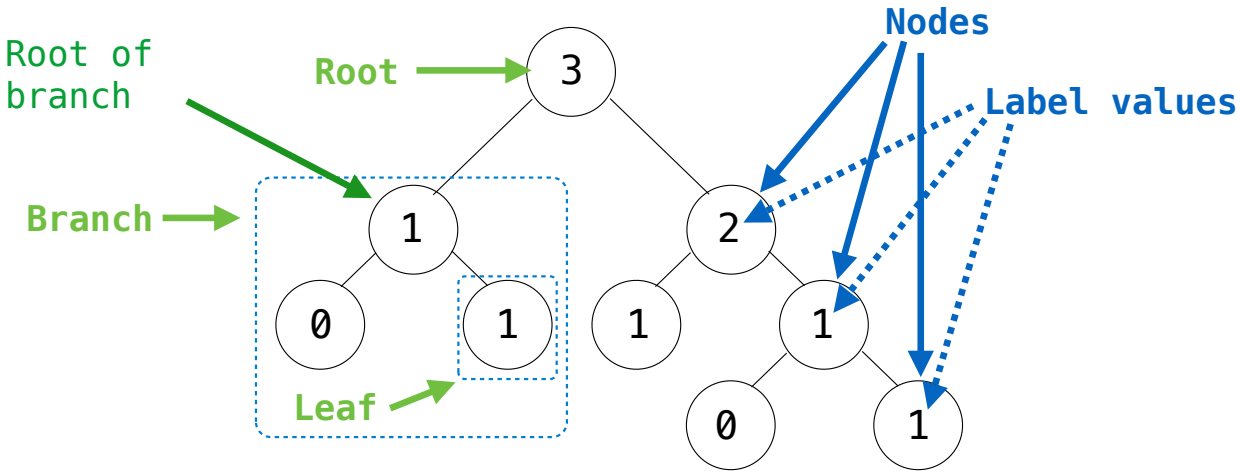


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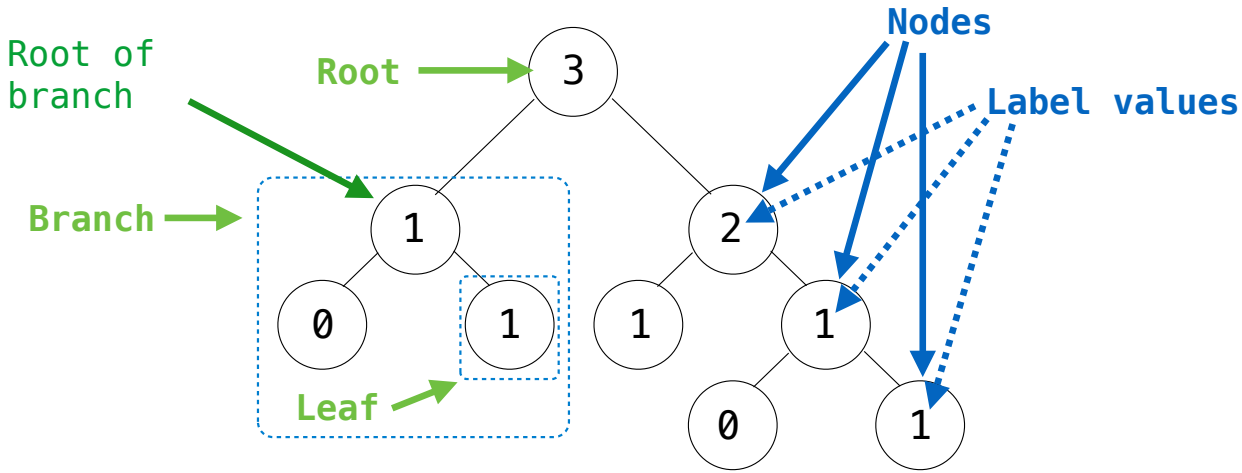


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One node can be the **parent/child** of another

*People often refer to values by their locations:* "*each parent is the sum of its children*"

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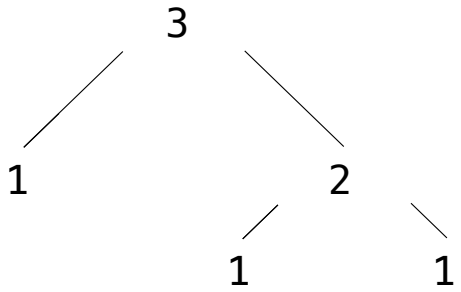

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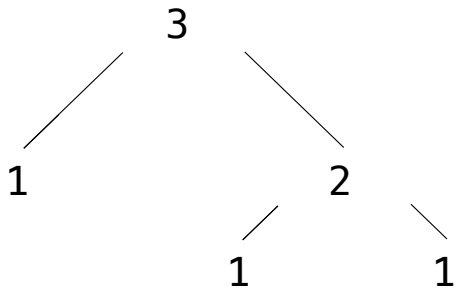


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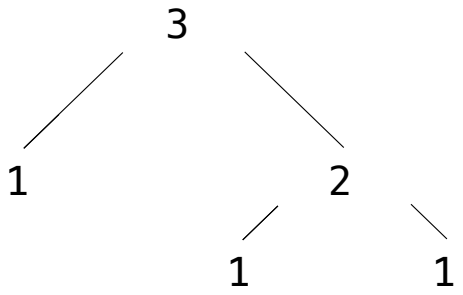


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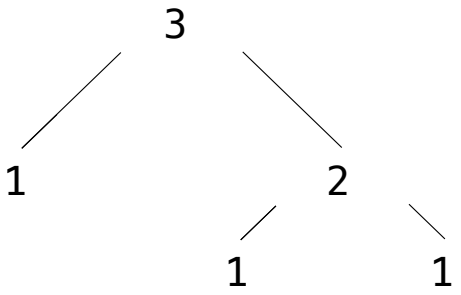


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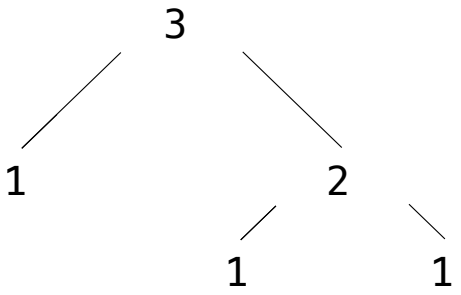


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def label(tree):
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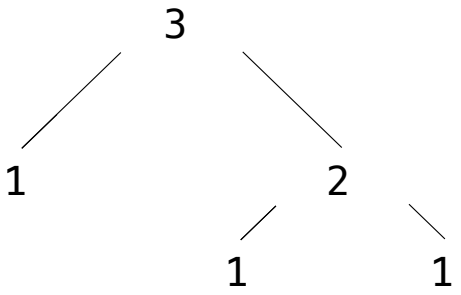


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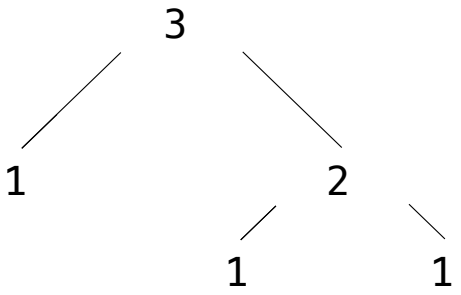


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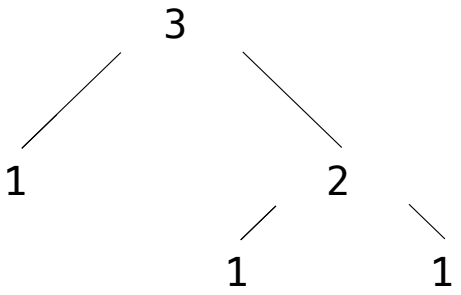


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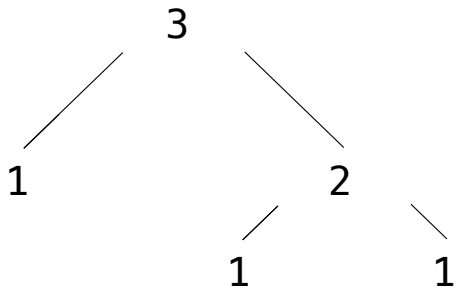


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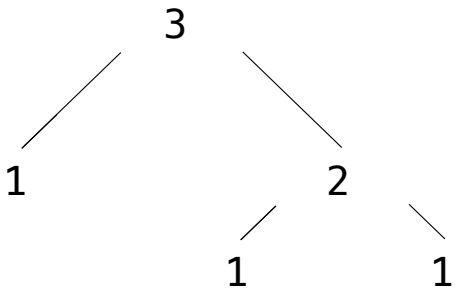


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Creates a list from a sequence of branches

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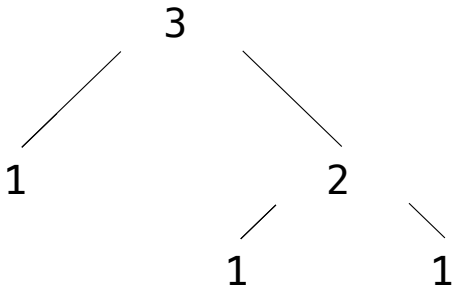


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Verifies the tree definition

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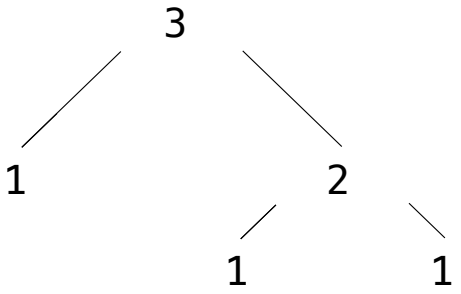


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def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
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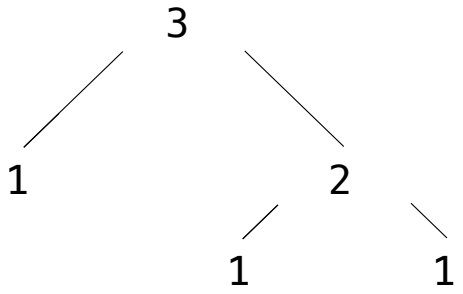


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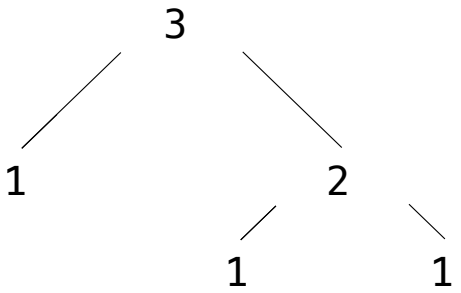
Verifies the tree definition

Creates a list from a sequence of branches

Verifies that tree is bound to a list

- A tree has a label value and a list of branches




```
>>> tree(3, [tree(1),
...          tree(2, [tree(1),
...                   tree(1)])])
[3, [1], [2, [1], [1]]]
```

# Implementing the Tree Abstraction

```
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)

def label(tree):
    return tree[0]

def branches(tree):
    return tree[1:]

def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True
```

Verifies the tree definition

Creates a list from a sequence of branches

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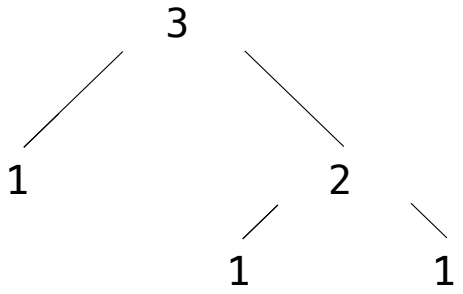


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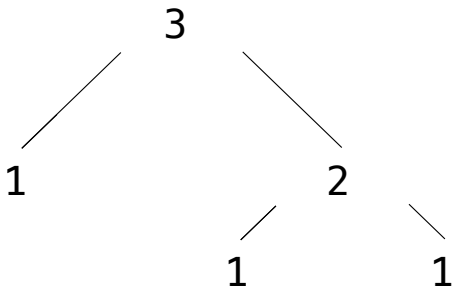


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(Demo)

# Tree Processing

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(Demo)

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```
branches(tree)
leaves(tree)
[branches(b) for b in branches(tree)]
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        return sum(List of leaves for each branch, [])
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# Example: Printing Trees

(Demo)